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Identification of Flexible Joints in Vehicle Structures

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A method for identification of flexible joints in vehicle structures is presented. The method identifies the most important parameters of a joint model and estimates their values by using displacement measurements of the overall structure. Thus, the method can be used to develop simple, design-oriented joint models that are suitable for preliminary structural design. The viability of the proposed method is demonstrated by applying it to plane frame structures with flexible joints.

Nomenclature

$\boldsymbol{b}$ = vector of parameter estimates
$\boldsymbol{b}^j$ = vector of parameter estimates obtained by using the jth data subset
$\boldsymbol{C}$ = covariance matrix
$\boldsymbol{C}_b^j$ = covariance matrix of parameters obtained by the jth data subset
$\boldsymbol{f}$ = force vector
$\boldsymbol{K}$ = global stiffness matrix
$\boldsymbol{K}_J$ = stiffness matrix of a joint
$\boldsymbol{K}_R$ = stiffness matrix of the vehicle structure without joints
$\boldsymbol{u}$ = vector of displacements
$\boldsymbol{u}_m$ = vector of measured displacements
$\hat{\boldsymbol{u}}$ = vector of estimated displacements
$\boldsymbol{x}$ = vector of regressor variables
$\boldsymbol{Z}$ = matrix of sensitivity derivatives of the displacements with respect to the parameters of the joints
$\boldsymbol{\beta}$ = vector of parameters of a joint model
γ = the vector of transformed joint parameters in principal direction analysis
ε = vector of measurement errors
$\binom{n}{m}$ = combinations of n objects taken m at a time

Subscripts
b = vector of parameter estimates
ε = vector of measurement errors

I. Introduction

VERY detailed finite element models of vehicle structures are used to predict their static and dynamic structural response. However, for preliminary design, it is more reasonable to employ simpler, design-oriented models. Such models should be inexpensive to analyze so that many design iterations can be executed. These models should be defined in terms of a small set of design variables that further alleviates the designer's job.

One of the major difficulties in developing and employing a design-oriented model of a vehicle structure is to have a simple and accurate model of the flexible joints at body connections. Although little is known on how to construct simple accurate models of such joints, it is widely accepted that the flexibility of the joints may, in some cases, dominate the response of vehicle structures.[1]

The following are some of the obstacles encountered in modeling the behavior of joints: 1) The joint branches may be flexibly connected in rotation in all three planes of the three dimensional space. 2) The rotations of the branches at different planes are coupled. 3) The branches do not rotate about the same point. 4) The behavior of the joints may be nonlinear even for small angles of rotation of its branches.

According to the aforementioned facts, it is difficult to model the behavior of joints. Consider, for example, a finite element model with $6n$ degrees of freedom, representing a joint with n branches. If the behavior of the joint is assumed to be linear, we need $(6n - 6) \times (6n - 5)/2$ independent parameters in order to define the stiffness matrix of this model.

The flexibility of joints has been studied by various researchers in several engineering fields. Chang[1] used torsional springs to model joints in automotive structures and demonstrated that their flexibility is important. Chon[2] proved the existence of upper and lower sensitivity thresholds for the parameters of a joint modeled by torsional springs. More specifically, he demonstrated that if a joint is too stiff or too flexible in rotation in a particular plane compared to the remaining structure, then it can be considered as rigid or free to rotate, respectively. This allows us to simplify a joint model by neglecting some of its parameters. Borowski[3] used translational and torsional springs to represent the flexible joints in car frames. He determined the unknown values of the spring stiffness parameters by tuning them so that theoretical predictions match dynamic test results. Hughes[4] developed models of the joints in transverse frames of ships. His models are two dimensional, and besides the joint flexibility, they also account for the rigid behavior of the bracketed branches in the vicinity of joints. The joint branches rotate about the geometrical center of the joint and they are connected by torsional springs. Hughes demonstrated that standard frame analysis, which treats joints as rigid, leads to errors in the deflection and bending moment, which might reach 50% of the corresponding actual values. Garstecki[5] studied optimal design of joints in linear elastic beams and frames. Ioannidis and Kounadis[6] studied the effect of joint flexibility on the nonlinear buckling load of metal portal frames.

Since our work employs system identification to identify the joint models and simplify them, we briefly review some recent studies in this area. The objective of system identification is to identify a model that represents the behavior of a given system with acceptable accuracy. In system identification we employ estimation methods, deterministic or probabilistic, in order to determine the parameters of the model.[7]

Baruch[8] and Berman and Nagy[9] developed a system identification procedure to determine the parameters of a structural system from dynamic test results. They determined the parameters by identifying a set of minimum changes in the original stiffness or mass matrices so that the eigensolution exactly agrees with test measurements. However, only in Baruch's method the rigid body modes of the identified system remain the same as those of the original system. Usually, it is difficult to mea-

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sure the mode shapes of a vibrating system. Methods have been recently introduced for identifying the mode shapes from dynamic response measurements.[10] Butkunas[11] successfully identified regions in the models of automotive structures that should be changed so that finite element predictions exactly agree with experimental results. In this approach, the overall car model was calibrated so that theoretically predicted natural frequencies and mode shapes exactly match measurements. In the aforementioned studies there are more unknown parameters than measurements, which allows us to exactly match predictions with measurements.

A probabilistic approach for system identification for nonlinear systems, which employs a recursive filtering algorithm, was proposed by Yun and Shinozuka.[12] Collins[13] developed a probabilistic identification method that uses measurements of mode shapes and natural frequencies to estimate the parameters of a linear dynamic system. This method essentially consists of repeated application of a sequential linearization technique (see, for example, Draper and Smith[14] and Bard[15]).

Recursive estimation[16] has also been used and found to be efficient and effective in determining the parameters of a model. A review on this type of estimation is done by Isenberg.[17] Allen and Martinez[18] integrated a commercial software package for finite element analysis (MSC/NASTRAN) and a computer code for Bayesian estimation of linear systems. Their computer code was used to successfully identify models for a truss structure and an electronic package.

A comparison between the deterministic approach used in Refs. 8–11 and probabilistic estimation leads to the conclusion that the latter is more reliable than the former. The following are some of the advantages of probabilistic estimation: 1) It accounts for measurement errors and gives more weight to more accurate measurements. 2) It provides measures of the errors in parameter estimates.

The objective of this work is to develop a method for identifying and simplifying joint models for vehicle structures. The basic idea is to estimate the model parameters and identify the important ones from the measured responses of the overall structure by employing probabilistic nonlinear estimation.[14,15] More specifically, joint models are identified by minimizing the discrepancy between theoretically predicted displacements and measured ones. In our opinion, the idea of using measurements of the overall car has the following advantages:

1) The difference between important parameters and unimportant ones is that only the former significantly affect the deflection of the vehicle body. Thus, the most appropriate way to exploit this fact and identify the important parameters is to examine the sensitivity of the displacements of the overall vehicle structure with respect to all parameters. The proposed method accounts for this sensitivity and thus it is expected to be more effective than methods that deal only with displacement measurements on isolated joints.

2) The flexible behavior of a joint connected to the other structural members of the vehicle may be considerably different from that of an isolated joint with unconstrained ends. Because of the coupling between the axial and torsional responses of the branches, the joint has an effective torsional stiffness that may be considerably higher than that of a joint isolated from the rest of the vehicle. The proposed method estimates these effective stiffnesses rather than measuring nonrepresentative values of the stiffness parameters as it is done in experiments performed on isolated joints.

In this study, we neglect the effect of modeling error in describing the behavior of the overall structure. This type of error might lead to significantly biased parameter estimates. However, to the best of our knowledge, there is no estimation method that takes into account this type of error. The effect of modeling error and the ability of the joint model to represent the behavior of the actual joint can be assessed by comparing the response of vehicle model, which includes the identified joint models against that of the actual structure under loading conditions that have not been used in estimation.

First, we develop a two-dimensional finite element model for a flexible joint. It is assumed that the joint behavior is linear and that its branches rotate about its geometric center. The spring stiffnesses are the joint parameters to be estimated. Then, we formulate the problem by defining the nonlinear estimation model relating the vehicle static response to the joint parameters. We also present a methodology for estimating these parameters and for simplifying the joint models. Finally, this methodology is demonstrated by applying it to identify the joints of an automotive frame structure.

II. Methods

A. Joint Parameterization

A flexible joint is usually modeled using torsional springs that constrain the relative rotations between its branches.[1,2,19] In this study, a flexible joint with n branches in a plane is modeled by $\binom{n}{2}$ torsional springs. The stiffnesses of these springs form a set of independent parameters that define the joint model. In modeling the joint, the following assumptions are made: 1) The behavior of the joint is linear elastic. 2) The joint branches are rigidly connected in translation. 3) The joint branches rotate around the same point, which is called the center of the joint.

A finite element model can be developed for an n-branch joint by considering n nodes located at its center. Thus, we have $3n$ degrees of freedom (DOF), which correspond to two translations and one rotation for each node. According to the second assumption, the translational DOF of all connecting nodes are the same. Therefore, we have $(n + 2)$ DOF for an n-branch joint in a plane. The stiffness matrix corresponding to this joint model is an $(n + 2) \times (n + 2)$ matrix, whose first two rows and columns correspond to horizontal and vertical deflections and contain zeros. The remaining rows and columns correspond to the rotations of the n nodes. For example, the stiffness matrix $\boldsymbol{K}_J$ of the 3-branch joint element in Fig. 1 is as follows:

$$\boldsymbol{K}_J = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_1 + \beta_3 & -\beta_1 & -\beta_3 \\ 0 & 0 & -\beta_1 & \beta_1 + \beta_2 & -\beta_2 \\ 0 & 0 & -\beta_3 & -\beta_2 & \beta_2 + \beta_3 \end{bmatrix} \tag{1}$$

Each of the n nodes of the joint model is connected to the end of the corresponding beam of the remaining structure containing the joint.

Thus, we can form the global stiffness matrix $\boldsymbol{K}$ of a structure with m flexible joints by assembling the stiffness matrices of the joints, $\boldsymbol{K}_{Ji}$ $(i = 1, \ldots, m)$, with that of the remaining structure without joints, $\boldsymbol{K}_R$:

$$\boldsymbol{K} = \sum_{i=1}^{m} \boldsymbol{K}_{Ji} + \boldsymbol{K}_R \tag{2}$$

The resulting model accounts for the flexibility of the joints.

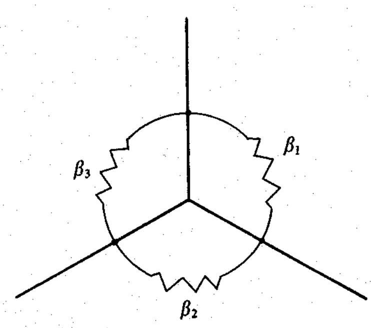

Fig. 1 Model of a flexible joint with three branches in a plane.

B. Nonlinear Estimation Model

The estimation model in this paper is nonlinear. More specifically, the analytically predicted displacements of a structure with flexible joints are related to the stiffness parameters according to the following nonlinear equation:

$$u = K(\beta, x)^{-1} f \tag{3}$$

where u is a displacement vector, f is a force vector, and K is a global stiffness matrix whose elements are functions of the vector of the joint stiffness parameters, β, and the vector of regressor variables, x, which defines the magnitudes and positions of applied loads and the boundary conditions.

There are discrepancies between analytical and experimental displacements, which are due to the following: 1) errors in measuring displacements; 2) errors in modeling the joints; and 3) errors in modeling the remaining structure without the joints.

In this study, we neglect the latter two types of error. The measured displacement will be written as

$$u_m = K(\beta, x)^{-1} f + \varepsilon \tag{4}$$

where u_m is the vector of measured displacements and ε is the vector of measurement errors. It is reasonable to assume that the elements of ε are zero mean Gaussian random variables. However, they are not identically distributed because the errors are not the same for all measurements. The statistics of ε are defined by its covariance matrix C_ε. The diagonal entries of C_ε are the variances of the measurement errors. Each nondiagonal entry is the covariance of the measurement errors in the displacements corresponding to the position of the entry.

C. Estimation

1. Classical (Batch) Estimation

The joint parameters are identified by using a procedure that is called nonlinear weighted estimation.[14–16,20] In nonlinear weighted estimation, the vector of parameter estimates b is determined by minimizing the weighted sum of squares of errors between predicted and measured displacements:

$$(u_m - \hat{u})^T C_\varepsilon^{-1} (u_m - \hat{u}) \tag{5}$$

where u_m is the vector of measured displacements and $\hat{u}$ is the vector of the estimated displacements corresponding to b. The inverse of the covariance matrix of measurement errors, C_ε^{-1}, is introduced in Eq. (5) in order to account for the level of accuracy associated with each measurement and weight measurements accordingly.

We use the Gauss-Newton method, also known as the successive linearization method, to solve the nonlinear estimation problem.[14,15] By taking a linear Taylor series expansion of the model in Eq. (3) about some initial estimate of the parameter vector b_i, and by setting the derivative of the weighted error sum of squares in Eq. (5) with respect to b equal to zero, the vector of improved parameter estimates, b_{i+1}, can be found by the following:

$$b_{i+1} = b_i + (Z^T C_\varepsilon^{-1} Z)^{-1} Z^T C_\varepsilon^{-1} (u_m - \hat{u}) \tag{6}$$

where both $\hat{u}$ and Z are calculated for $\beta = b_i$.[14,20] Z is the matrix of sensitivity derivatives whose jth column, $\partial u / \partial b_j$, is the derivative of the displacement vector u with respect to the jth parameter b_j, and is found by the Direct method[21]:

$$K \frac{\partial u}{\partial b_j} = \frac{\partial f}{\partial b_j} - \frac{\partial K}{\partial b_j} u \tag{7}$$

Equation (6) can be used to define an iterative procedure for estimating b. We start with an initial guess for the vector of parameter estimates b_0. Equation (6) can be used to obtain a sequence of vectors of parameter estimates, $b_0, b_1, \ldots, b_n$. In most of the cases this sequence converges to the vector of the actual values of parameters β. The above iterative procedure is stopped according to some convergence criterion.

The convergence of the Gauss-Newton method is not robust. Fortunately, the convergence can be improved by using a step factor.[20,21] to restrain the change in the vector of parameter estimates at every iteration. This idea is implemented by using the following equation for the updated vector of parameter estimates b_{i+1}:

$$b_{i+1} = b_i + \lambda \Delta b_i \tag{8}$$

where λ is a step factor and Δb_i is a vector of parameter increments determined at the ith iteration stage without using a step factor. Δb_i is obtained from Eq. (6) as the difference between b_{i+1} and b_i. We start with $\lambda = 1$ at the beginning of each iteration. If the objective function in Eq. (5) corresponding to b_{i+1} is larger than the value corresponding to b_i, we reduce λ by half. We keep evaluating the objective function at the corresponding b_{i+1} and reducing λ by half until the objective function becomes smaller than the value corresponding to b_i.

The covariance matrix of parameters is calculated from

$$C_b = (Z^T C_\varepsilon^{-1} Z)^{-1} \tag{9}$$

where Z is calculated at the converged value of the vector of parameter estimates b.

2. Bayesian (Recursive) Estimation

In Bayesian estimation, we divide a large set of measurements into smaller subsets and estimate the parameters using each data subset successively. Every time we consider a new data subset, we use the prior parameter estimates and their covariance matrix to update the parameter estimates and their covariance matrix.[17,18] For example, consider the $(j + 1)$th data subset with the prior vector of parameter estimates b^j and their covariance matrix C_b^j which have been found by using the jth data subset. The updated vector of parameter estimates b^{j+1} is determined so that it minimizes the weighted error sum of squares plus the difference between the prior vector of parameter estimates b^j and the updated one b^{j+1}, weighted by the inverse of the covariance matrix of the prior parameter estimates $[C_b^j]^{-1}$:

$$(u_m - \hat{u})^T [C_\varepsilon]^{-1} (u_m - \hat{u}) + (b^j - b^{j+1})^T [C_b^j]^{-1} (b^j - b^{j+1}) \tag{10}$$

where u_m is the vector of measured displacements in the $(j + 1)$th data subset and $\hat{u}$ is the vector of the corresponding estimates of these displacements. In Bayesian estimation, the iterative procedure for the updated vector of parameter estimates is obtained from the following equation[17,18]:

$$b_{i+1}^{j+1} = b^j + \{[C_b^j]^{-1} + Z^T C_\varepsilon^{-1} Z\}^{-1} Z^T C_\varepsilon^{-1} \times \{u_m - u - Z(b^j - b_i^{j+1})\} \tag{11}$$

Equation (11) is used to update the estimates of parameters by using the $(j + 1)$th data subset, the prior estimates, b^j, and their covariance matrix, C_b^j. Superscripts j and $(j + 1)$ in Eqs. (10) and (11) denote the data subset number. Subscripts i and $(i + 1)$ indicate the iteration number in the Gauss-Newton method (Fig. 2). Both u and Z are calculated for $\beta = b_i^{j+1}$. The covariance matrix of the updated parameter estimates is calculated from the following equation:

$$C_b^{j+1} = ([C_b^j]^{-1} + Z^T [C_\varepsilon]^{-1} Z)^{-1} \tag{12}$$

If the values of the prior parameter estimates are unknown at the beginning of estimation, we start with very large values in the diagonal elements and zeros in the off-diagonal elements for C_b^j in Eqs. (11) and (12).

When the estimation model is linear with respect to the parameters, Bayesian estimation gives the same results as a pro-

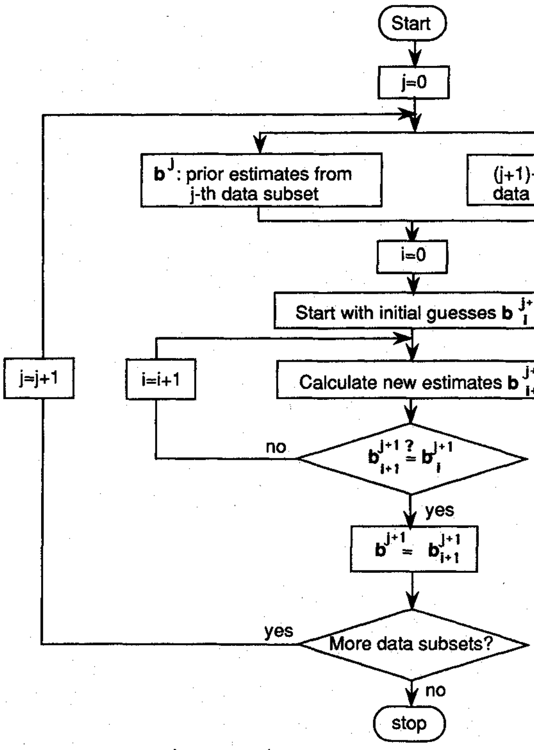

Fig. 2 Schematic diagram for Bayesian estimation: Superscripts *j* and *j* + 1 denote the number of data subsets (outer loop); subscripts *i* and *i* + 1 denote the iteration number in successive linearization method (inner loop).

cedure that employs classical estimation and estimates the parameters by considering all of the data subsets simultaneously. When the estimation model is nonlinear with respect to the parameters, Bayesian estimation gives results that are approximately equal to those from classical estimation.

Finally, another advantage of Bayesian estimation is that it allows us to improve the efficiency of estimation by designing experiments properly. For example, consider the case that some parameters are poorly estimated because the measurements are not sensitive to these parameters. By using these results, we can determine a new subset of measurements that are sensitive to the aforementioned parameters and update the estimates by using Bayesian estimation. This procedure will be demonstrated in Sec. III.B.

D. Practical Considerations

The response of a structure may be insensitive to some stiffness parameters. This usually occurs when the joint is either too stiff or too flexible in some directions compared to the rest of the structure.[2] For each parameter, there are two values called lower and upper sensitivity thresholds. If the value of a parameter is out of the range defined by the lower and upper sensitivity thresholds, then it does not affect the response of the structure. Depending on the specific case, the joint model can be simplified by considering that the joint is infinitely flexible or stiff in some particular direction.

In deciding which parameters are unimportant, we may use the covariance matrix of the parameter estimates and/or their confidence contours. However, the estimates of some parameters might be poor not only because these parameters are unimportant but also because the experiments were not well-designed. We can improve these estimates by taking additional measurements that are sensitive to these parameters and by applying Bayesian estimation. If the standard deviation of the estimates reduces significantly, then these parameters are important and they should be retained in the model. Otherwise, we can simplify the model by discarding them. In this section, we describe some tools that can be used to identify the unimportant parameters. After discarding unimportant parameters, we use the lack-of-fit test to check if the simplified model for the joint fits the data.

The following statistical tools are exactly true for linear estimation problems only. In nonlinear estimation, which is the case of this paper, these tools yield approximate results.

1. Covariance Matrix of Parameters

The covariance matrix of the parameter estimates C_b is obtained from Eq. (9) or Eq. (12). The diagonal elements of this matrix are the variances of the corresponding parameter estimates. These variances indicate the reliability of the corresponding estimates; the larger the variance, the poorer the estimate. Practically, we assess the reliability of a parameter estimate by using the coefficient of variation ρ_i which is given by

$$\rho_i = \sigma_{bi}/b_i \tag{13}$$

where σ_{bi} is the standard deviation of the *i*th parameter and b_i is its estimate. A large coefficient of variation means that either the parameter is unimportant or the measurements are not sensitive to this parameter.

2. Confidence Contours

A confidence contour is a region in the parameter space, centered around the estimated values of parameters, such that the actual values of the parameters fall within this region with a prescribed probability.[14] The smaller the size of the confidence contour in a particular direction, the higher the reliability of the estimate for the corresponding parameter.

If the measurement errors are normally distributed, then the probability distribution of the estimates of parameters is approximately normal. Therefore, $L(\boldsymbol{b}) = (\boldsymbol{\beta} - \boldsymbol{b})^T \boldsymbol{Z}^T \boldsymbol{C}_\varepsilon^{-1} \boldsymbol{Z}(\boldsymbol{\beta} - \boldsymbol{b})$ follows the χ^2-distribution with p degrees of freedom, where p is the number of parameters. The weighted error sum of squares $S(\boldsymbol{b}) = (\boldsymbol{u}_m - \hat{\boldsymbol{u}})^T \boldsymbol{C}_\varepsilon^{-1}(\boldsymbol{u}_m - \hat{\boldsymbol{u}})$ follows the χ^2-distribution with $(n - p)$ DOF, where n is the number of measurements. Then, the ratio

$$\frac{L(\boldsymbol{b})/p}{S(\boldsymbol{b})/(n-p)}$$

follows the F-distribution with p and $(n - p)$ DOF. The vector of parameters $\boldsymbol{\beta}$ which satisfies

$$(\boldsymbol{\beta} - \boldsymbol{b})^T \boldsymbol{Z}^T \boldsymbol{C}_\varepsilon^{-1} \boldsymbol{Z}(\boldsymbol{\beta} - \boldsymbol{b}) \le \frac{p}{n-p} (\boldsymbol{u}_m - \hat{\boldsymbol{u}})^T \times \boldsymbol{C}_\varepsilon^{-1}(\boldsymbol{u}_m - \hat{\boldsymbol{u}})\, F(p, n-p, 1-q) \tag{14}$$

gives an approximate confidence contour, where $F(p, n - p, 1 - q)$ denotes the F distribution with p and $(n - p)$ DOF at the $(1 - q)$ probability level. The probability that the actual values of parameters lie within the contour is $1 - q$.

In practice, we plot sectional drawings of the confidence contour in planes that correspond to each pair of parameters β_i and β_j. All other parameters are equal to their estimated values $\beta_k = b_k (k \neq i, j)$ in each of these planes. If all sectional drawings in different planes are very long in the direction that corresponds to a particular parameter, then either this parameter is unimportant or the experimental measurements are insensitive to this parameter.

3. Principal Directions

Principal direction analysis helps us to identify the important parameters. In this procedure, we find a vector of transformed parameters which is the projection of the vector of parameter estimates on the coordinate system defined by the eigenvectors of the covariance matrix of the regressor variables.[22] This matrix is the inverse of the covariance matrix of the parameter estimates. The transformed parameters, which are linear combinations of original parameters, are statistically independent.

Each of these parameters corresponds to an eigenvalue and an eigenvector of the covariance matrix of regressor variables. It can be shown that each eigenvalue of the covariance matrix of regressor variables is equal to the change in the weighted error sum of squares due to a unit change in the value of the corresponding transformed parameter.[14] Thus, the magnitude of an eigenvalue is a measure of the relative importance of the corresponding transformed parameter; the larger the eigenvalue, the more important the corresponding transformed parameter. The magnitudes of elements in the corresponding eigenvector indicate the relative contributions of the original parameters to the transformed parameters.

Principal direction analysis allows us to identify and discard unimportant parameters. An original parameter is important if it mostly participates in eigenvectors whose corresponding eigenvalues are large compared to the smallest eigenvalue. On the other hand, an original parameter is unimportant if it only participates in eigenvectors whose corresponding eigenvalues are small compared to the largest eigenvalue.

Principal direction analysis can also be used to reduce the number of parameters by replacing them with a smaller number of new parameters that are linear combinations of the original ones. These linear combinations are defined by the eigenvectors that correspond to the largest eigenvalues. The procedure is demonstrated in Sec. III.D.

4. Lack-of-Fit Test

The lack-of-fit test can be used to check if a joint model fits the measured data, which is equivalent to checking if all of the important parameters have been included in the model.[14] In the lack-of-fit test, we compare the pure error due to the noise in the measurements with the error due to the lack of fit, which is the discrepancy between theoretically predicted displacements and measured ones. If the latter is significantly larger than the former, then the discrepancy between predictions and measurements cannot be attributed to the randomness involved in the experiment. In that case, the lack-of-fit test is failed. If the model passes the lack-of-fit test, there is no reason to suspect that the model is incomplete. We can test whether a reduced model, which is obtained after discarding unimportant parameters, fits the data by performing the lack-of-fit test.

The lack-of-fit test requires repeated measurements that are taken by repeating the experiment while keeping the values of all regressor variables unchanged. Both the pure error and the error due to the lack of fit, weighted by the inverse of the covariance matrix of measurement errors, follow the χ^2-distributions. Then, the ratio of the pure and the lack-of-fit errors divided by the corresponding degrees of freedom follows the F distribution:

$$\frac{(\hat{\boldsymbol{u}} - \bar{\boldsymbol{u}})^T \boldsymbol{C}_\varepsilon^{-1} (\hat{\boldsymbol{u}} - \bar{\boldsymbol{u}})/(n - p - n_e)}{(\boldsymbol{u}_m - \bar{\boldsymbol{u}})^T \boldsymbol{C}_\varepsilon^{-1} (\boldsymbol{u}_m - \bar{\boldsymbol{u}})/n_e} \sim F(n - p - n_e, n_e, 1 - q) \qquad (15)$$

where $\bar{\boldsymbol{u}}$ is a vector whose elements are average displacements from each set of repeated measurements, n is the number of total measurements, p is the number of parameters, m is the number of different sets of values of regressor variables, $n_e = n - m = \sum_{j=1}^{m} n_j - m$ denotes the number of degrees of freedom of the pure error, $(1 - q)$ is the probability level, and n_j is the number of repeated measurements at the jth set of values of regressor variables. When the ratio in Eq. (15) is smaller than $F(n - p - n_e, n_e, 1 - q)$, the model passes the lack-of-fit test at a probability level $(1 - q)$.

III. Examples

The following procedure was used to test and illustrate the proposed method for the identification of joints. We considered a finite element model of a car body with flexible joints. Loads were applied to the car body and the resulting displacements were calculated by finite element analysis. To simulate the errors in measurements, we generated random numbers from the assumed probability distribution of the measurement errors. The calculated displacements were contaminated by adding the generated numbers to them. Thus, the resulting sums simulated the measured displacements. Then, we assumed that the values of the joint parameters are unknown and used the estimation method and the simulated measurements to estimate them. The above procedure was applied to four examples. Since we considered static displacements only, we assumed that the standard deviations of the measurement errors were 1% of exact displacements in most of the examples. We investigated the effect of error in measurements and the number of measurements in Sec. III.C. The car model that was used in the examples is shown in Fig. 3. The material and geometrical properties of the car structure are given in Table 1.

A. Classical Estimation and Simplification of a Joint Model

Figure 3 shows a car model with four stiffness parameters. The values of the parameters are $\beta_1 = 1.13 \times 10^8$, $\beta_2 = 5.65 \times 10^9$, $\beta_3 = 1.13 \times 10^8$, and $\beta_4 = 1.13 \times 10^7$ N·mm/rad. We measured displacements u_1, v_1, u_2, v_2, u_3, u_5 with load L_1 applied. The magnitude of L_1 was 4448 N. Each displacement was measured five times under which the structure was subjected to the same load. Thus, the total number of measurements was 30. The errors from repeated measurements were assumed to be Gaussian, statistically independent, random variables. The parameters were estimated by using the above measurements. The estimated model was also tested by performing the lack-of-fit test.

We summarize the results of classical estimation and principal direction analysis in Table 2. Figure 4 depicts the sectional drawing of the confidence contour in the plane of β_1 and β_2, which is the intersection of the hyper ellipsoid representing the confidence contour of the parameter estimates with the plane of $\beta_3 = b_3$ and $\beta_4 = b_4$. This confidence contour corresponds to a probability level of 0.95.

The following are observed from Table 2 and Fig. 4:

1) The estimate of β_2 and its coefficient of variation are large compared to those of the other parameters.

2) In principal direction analysis, the smallest eigenvalue is almost zero. Only β_2 contributes to the corresponding eigenvector. Furthermore, β_2 does not contribute significantly to the

Table 1 Material and geometric properties of a car structure in examples[a]

Part	moment of inertia I, mm^4	area A, mm^2
Shortgun	1.2971×10^7	9.4224×10^3
Hinge	1.2971×10^7	9.4224×10^3
Rocker	1.2051×10^6	1.1778×10^3
Roof	3.6503×10^5	9.0116×10^2
Windshield	3.2662×10^5	6.0077×10^2
Center pillar	8.6060×10^5	1.0467×10^3
Rear pillar	4.3030×10^5	5.2335×10^2

[a] Young's modulus: 2.0683×10^5 N/mm^2.

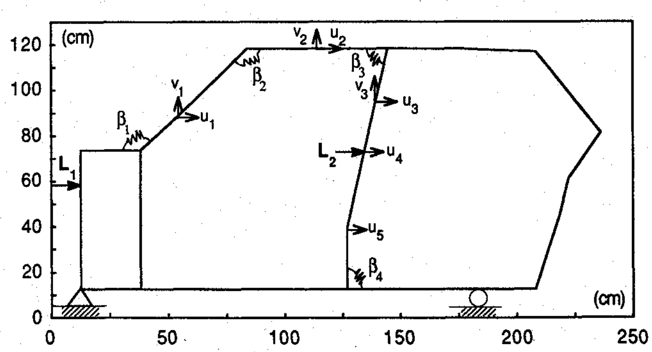

Fig. 3 Car model (examples III.A–C).

Table 2 Classical estimation using a full model (β_1, β_2, β_3, β_4)—example III.A

Classical estimation[a]			
i	Exact value of parameter β_i, N · mm/rad	Estimate of parameter b_i, N · mm/rad	Coefficient of variation ρ_i
1	1.13×10^8	1.092×10^8	0.08
2	5.65×10^9	4.226×10^9	2.62
3	1.13×10^8	1.040×10^8	0.07
4	1.13×10^7	1.014×10^7	0.27

Principal direction analysis

j	Eigenvalue λ_j	Eigenvector e_j			
1	0.15×10^{-7}	−0.19	0.00	0.18	0.97
2	0.89×10^{-9}	−0.56	0.00	0.79	−0.26
3	0.11×10^{-9}	0.81	0.00	0.59	0.05
4	0.10×10^{-15}	0.00	1.00	0.00	0.00

[a]Weighted error sum of squares = 5.3813×10^{-4}.

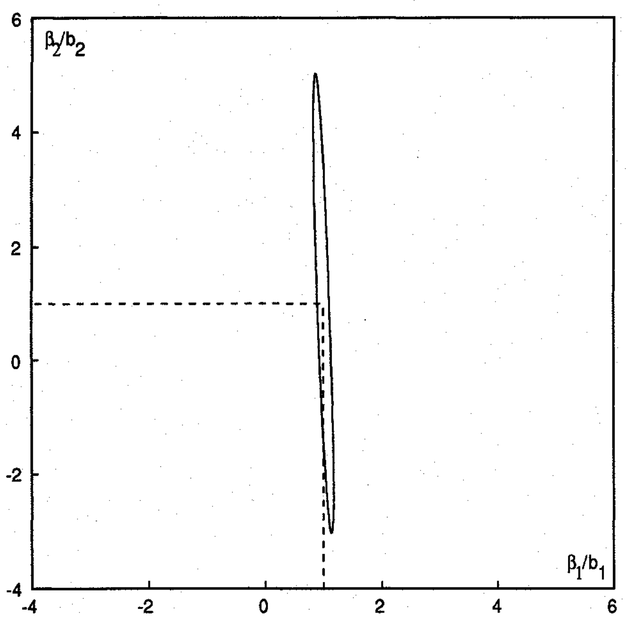

Fig. 4 Sectional drawing of confidence contour in plane of β_1 and β_2 (example III.A).

first three eigenvectors that correspond to the three largest eigenvalues.

3) The sectional drawing of the confidence contour in the plane of β_1 and β_2 is a thin ellipse whose long axis is almost parallel to the direction of β_2. The other sectional drawings of the confidence contour are also thin ellipses in the direction of β_2. Actually, the hyper ellipsoidal confidence contour in a multidimensional space is very long in the direction of β_2.

4) The estimate of β_2 is large compared to the estimates of other parameters, which indicates that the actual value of β_2 exceeds the upper sensitivity threshold. We can calculate the weighted error sum of squares with either $\beta_2 = 0$ or $\beta_2 = \infty$. The weighted error sum of squares is smaller when $\beta_2 = \infty$, which means that the joint is rigid rather than free to rotate.

Based on the preceding observations and the fact that the model in Table 2 passed the lack-of-fit test, it appears that the model can be simplified by discarding β_2.

Table 3 summarizes the results of classical estimation and principal direction analysis for a reduced model containing only three parameters β_1, β_3, and β_4. By comparing Tables 2 and 3, we observe that the reduced model fits the measurements equally well as the full model. Indeed, the weighted error sum of squares are almost equal for both models. Furthermore, the estimated values of β_1, β_3, and β_4 in the reduced model are closer to the actual values than in the full model. Therefore,

Table 3 Classical estimation using a reduced model (β_1, β_3, β_4)—example III.A

Classical estimation[a]			
i	Exact value of parameter β_i, N · mm/rad	Estimate of parameter b_i, N · mm/rad	Coefficient of variation ρ_i
1	1.13×10^8	1.094×10^8	0.08
3	1.13×10^8	1.053×10^8	0.07
4	1.13×10^7	1.101×10^7	0.13

Principal direction analysis

j	Eigenvalue λ_j	Eigenvector e_j		
1	0.15×10^{-7}	−0.19	0.18	0.97
2	0.88×10^{-9}	−0.56	0.79	−0.25
3	0.11×10^{-9}	0.81	0.58	0.05

[a]Weighted error sum of squares = 5.4188×10^{-4}.

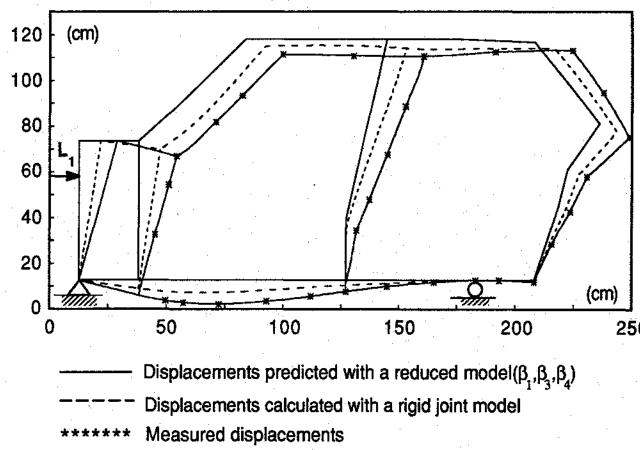

Fig. 5 Predicted displacements with a reduced model containing three stiffness parameters, β_1, β_3, and β_4 (example III.A).

the reduced model is sufficiently accurate for predicting the response of the car body.

Finally, we performed the lack-of-fit test in Eq. (15) to verify that the new model with β_1, β_3, and β_4 includes all of the important parameters. The probability level used in this test is 0.95. The left-hand side of Eq. (15) was found to be equal to 0.11, which is less than the right-hand side, $F(3, 24, 0.95) = 3.01$. Therefore, there is no reason to reject the hypothesis that the reduced model fits the measurements.

In Fig. 5, we plot the displacements predicted with the reduced model, the displacements calculated with a rigid-joint model, and the measured displacements. It is also observed that there are large discrepancies between the displacements calculated with a rigid-joint model and the measurements, which demonstrates the importance of considering the flexibility of joints. The displacements predicted by the reduced model are in very good agreement with the measurements.

B. Bayesian Estimation for Improving the Parameter Estimates

We estimated the joint parameters of the same car body as in the previous example by measuring the displacements u_1, v_1, u_2, v_2, u_3, u_5 once with load L_1 applied. We summarize the results of estimation and principal direction analysis in Table 4. As in the previous example, we observe from Table 4 that the measured displacements are not sensitive to β_2. However, we cannot draw any definite conclusion regarding β_4 because, although the coefficient of variation of β_4 is large compared with those of β_1 and β_3, principal direction analysis indicates that β_4 is important.

To decide whether β_4 is important, we took some additional new measurements that were expected to be sensitive to β_4, and updated the prior parameter estimates in Table 4 by applying Bayesian estimation. Thus, we measured another set of

Table 4 Classical estimation using six measurements from load L_1—example III.B

	Classical estimation[a]				
i	Exact value of parameter β_i, N·mm/rad	Estimate of parameter b_i, N·mm/rad		Coefficient of variation ρ_i	
1	1.13×10^8	1.142×10^8		0.18	
2	5.65×10^9	1.000×10^{14}[b]		——[c]	
3	1.13×10^8	1.216×10^8		0.16	
4	1.13×10^7	1.600×10^7		0.40	
	Principal direction analysis				
j	Eigenvalue λ_j	Eigenvector e_j			
1	0.27×10^{-8}	−0.19	0.00	0.16	0.97
2	0.15×10^{-9}	−0.58	0.00	0.77	−0.24
3	0.20×10^{-10}	0.79	0.00	0.61	0.05
4	0.32×10^{-26}	0.00	1.00	0.00	0.00

[a]Weighted error sum of squares = 4.1555×10^{-5}.
[b]Upper side constraint for the parameter estimate.
[c]Very large coefficient of variation.

Table 5 Bayesian estimation using the prior estimates in Table 4 and six more measurements from load L_2—example III.B

	Bayesian estimation[a]				
i	Exact value of parameter β_i, N·mm/rad	Estimate of parameter b_i, N·mm/rad		Coefficient of variation ρ_i	
1	1.13×10^8	1.118×10^8		0.15	
2	5.65×10^9	1.722×10^9		0.52	
3	1.13×10^8	1.205×10^8		0.12	
4	1.13×10^7	1.545×10^7		0.16	
	Principal direction analysis				
j	Eigenvalue λ_j	Eigenvector e_j			
1	0.79×10^{-8}	−0.06	0.00	0.11	0.99
2	0.24×10^{-9}	−0.70	0.00	0.71	−0.12
3	0.51×10^{-10}	0.71	0.00	0.70	−0.04
4	0.21×10^{-13}	0.00	1.00	0.00	0.00

[a]Weighted error sum of squares = 8.1063×10^{-5}.

the displacements u_1, v_2, u_3, v_3, u_4, u_5 once, with load L_2 applied. The magnitude of L_2 was 1334.4 N. In Table 5, we summarize the results of principal direction analysis as well as the results of Bayesian estimation. By comparing Tables 4 and 5, we observe the following:

1) The reliability of the estimates of all parameters improved by using Bayesian estimation.

2) Principal direction analyses in both tables indicate that β_2 is not important.

3) The estimate of β_2 was dramatically changed and its coefficient of variation remained significantly larger than those of other parameters after applying Bayesian estimation. This indicates that the weighted error sum of squares and displacements are not sensitive to β_2.

4) The large value of the estimate of β_2 indicates that it exceeds the upper sensitivity threshold.

Considering all these observations, we conclude that β_2 can be discarded. On the other hand, the improvement in the reliability of the estimate of β_4 was significant. From the value of its coefficient of variation and the results of principal direction analysis, β_4 appears to be the most important parameter. Therefore, β_4 should be retained in the joint model.

C. Effect of Number of Measurements on Estimation Results

The objective of this example is to study the effect of measurement error and number of measurements on parameter estimates. For this purpose, we measured the displacements u_1, v_1, u_2, v_2, u_3, u_5 with load L_1 applied, to the same car body as in the previous examples (Fig. 3). The values of parameters were all equal to 1.13×10^8 N·mm/rad. This value is between the lower and upper sensitivity thresholds for each parameter. The magnitude of L_1 was 4448 N. The coefficients of variation of measurement errors were 5%. We took the above set of measurements once and also repeatedly for 10, 20, 30, 40, and 50 times. Thus, the total numbers of measurements were 6, 60, 120, 180, 240, and 300, respectively.

The estimate of β_1, which is normalized by its exact value, is plotted in Fig. 6 as a function of the number of measurements. Centered around the normalized estimate of β_1 is a band whose width is equal to two standard deviations of b_1. It is observed that the normalized estimate approaches the value of one and the width of the band decreases with the number of measurements increasing. Similar trends were observed for the other parameters.

D. Principal Direction Analysis for Simplifying a Joint Model

Figure 7 shows a three-branch joint of a car model in a plane, where three parameters β_1, β_2, and β_3 were assumed to be unknown. The values of these parameters were $\beta_1 = 1.13 \times 10^{12}$, $\beta_2 = 2.26 \times 10^8$, and $\beta_3 = 1.13 \times 10^8$ N·mm/rad. The value of β_1 was larger than the upper sensitivity threshold, which means that the two horizontal branches of the joint were practically rigidly connected. We measured displacements v_2, v_4, u_5 by applying load L_1 and v_1, v_3, u_6 by applying load L_2. The magnitudes of L_1 and L_2 were 3336 and 1334.4 N, respectively. Each displacement was measured four times.

We summarize the results of principal direction analysis in Table 6. For this example, the iteration procedure did not converge because of the strong correlation between parameters. However, principal direction analysis performed after a number of iterations shows how to simplify the model. More specifically, the first eigenvalue is at least 10^5 times larger than the other two. From the smallest eigenvalue and the corresponding eigenvector, it is clear that β_1 is not important. The second eigenvector, which is practically the difference be-

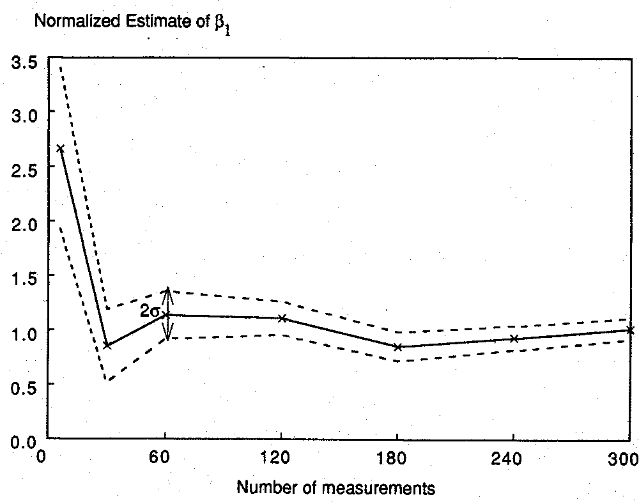

Fig. 6 Effect of number of measurements on the estimation results (example III.C).

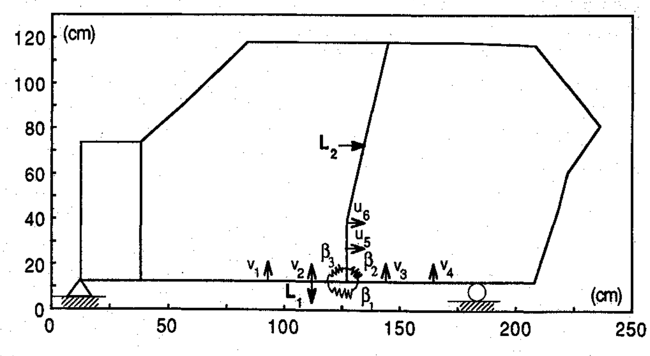

Fig. 7 Three-branch joint of a car model in a plane (example III.D).

Table 6 Classical estimation using a full model (β_1, β_2, β_3)—example III.D (principal direction analysis)

j	Eigenvalue λ_j	Eigenvector e_j		
1	0.14×10^{-7}	0.000	0.709	0.705
2	0.77×10^{-13}	0.002	−0.705	0.709
3	0.53×10^{-18}	1.000	0.002	−0.002

Note: Estimates are not available because of strong correlation between parameters.

tween β_2 and β_3, is also unimportant because the corresponding eigenvalue is small compared to the largest eigenvalue. The largest eigenvalue and the corresponding eigenvector indicate that β_2 and β_3 can be combined into one new parameter, which is their summation. This is the only important parameter. Thus, the joint model can be simplified by considering one parameter, $\beta_2 + \beta_3$, which restrains the rotation of the center pillar with respect to the rocker.

To validate the preceding conclusions, we considered a reduced-joint model containing only one parameter $\gamma = \beta_2 + \beta_3$, which restraints the rotation between the center pillar and the rocker. We found by using the same estimation procedure that $\hat{\gamma} = 3.408 \times 10^8$ N·mm/rad, which is almost equal to the sum of the actual values of β_2 and β_3. This model passed the lack-of-fit test, which indicates that there is no reason to suspect that it is underparameterized.

In conclusion, principal direction analysis is useful not only for discarding unimportant parameters but also for combining two or more important parameters into one, thus, reducing the number of parameters in the joint model.

IV. Conclusions

A methodology for modeling and identification of flexible joints in vehicle structures has been developed. It is assumed that the joint behavior is linear. The method utilizes the measured displacements of the overall structure. In the examples considered, it was demonstrated that 1) we can use nonlinear estimation techniques to estimate the parameters of flexible joint models from the measured displacements of the overall car body; 2) we have tools (covariance matrix, confidence contour, and principal direction analysis) to identify which parameters in the joint model are important; 3) Bayesian estimation permits the design of experiments so that the joint parameters can be estimated more efficiently and effectively than by classical estimation; and 4) we can improve the reliability of parameter estimates by taking more measurements repeatedly.

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